# SKEW SHOCK WAVE AT THE INTERFACE OF TWO MEDIA 

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Results of calculating an irregular regime of interaction of a skew shock wave (SSW) with interfaces of media as the acoustic impedance on the interaction surface increases are generalized to a class of materials and, accordingly, interfaces that is wider as compared to [1, 2].

In [1, 2], consideration was given to the interaction of an SSW with a flat immovable surface of contact (SC) between two media (gas-gas in [1] and gas-solid in [2]) within the limits of applicability of the hydrodynamic theory of shock waves. This problem can be generalized to a wider class of materials if we use as the equations of state for the two media only their most common forms: a polytrope for gases with the index $k$ in the upper half-plane and the index $m$ in the lower half-plane (Fig. 1) and a $D-U$ adiabat in the form $D=$ $a+b U$ for all other materials (the coefficients of the adiabat will be denoted by $a$ and $b$ in the upper half-plane and by $c$ and $d$ in the lower half-plane). As has already been mentioned, the impedance of the "upper" medium 1 (Fig. 1) must be higher than that of the "lower" medium (medium 2). As in [1, 2], for calculations we used a coordinate system connected with the point of contact (PC) between the SSW and the SC. In what follows, the medium described by the polytrope (a gas) will be denoted by PM , and the medium described by the $D-U$ adiabat, by AM.

Clearly, within the framework of the problem stated it is appropriate to consider three possible combinations of contacting media (PM-PM, PM-AM, and AM-AM) since the combination AM-PM is impracticable because of the fact that the impedance of an AM is always higher than the impedance of a PM. Let, in the initial state, medium 1 have the density $\rho_{1}$ and medium 2 have the density $\rho_{5}$, the pressure be the same on both sides of the contact and be equal to $p_{0}$, and $\rho_{1} c_{1}<\rho_{5} c_{5}$. The SSW velocity will be denoted by $D$; consequently, the flow velocity in the coordinate system connected with the PC is $q_{1}=D / \sin \varphi ; D>c_{1}$ and $D>c_{5}$. The parameters of medium 1 behind the SSW are determined by the Rankine-Hugoniot conditions and the equation of state; for a PM

$$
\begin{gather*}
p_{2}=\frac{2 \rho_{1} q_{1}^{2}}{k+1} \sin ^{2} \varphi-\frac{k-1}{k+1} p_{0}, \frac{\rho_{1}}{\rho_{2}}=\frac{k+1+(k-1) p_{2} / p_{0}}{k-1+(k+1) p_{2} / p_{0}}=K\left(p_{2} / p_{0}\right), \\
q_{2}=q_{1} \cos \varphi \sqrt{1+\tan ^{2} \varphi K^{2}\left(p_{2} / p_{0}\right)}, \quad \theta_{2}=\varphi-\arccos \sqrt{ }\left(\frac{1}{1+\tan ^{2} \varphi K^{2}\left(p_{2} / p_{0}\right)}\right), \tag{1a}
\end{gather*}
$$

and for an AM

$$
\begin{align*}
& p_{2}=p_{0}+\frac{\rho_{1} D^{2}}{b}\left(1-\frac{a}{D}\right), q_{2}=q_{1} \cos \varphi \sqrt{ }\left(1+\frac{\tan ^{2} \varphi}{b^{2}}\left(b-1+\frac{a}{D}\right)^{2}\right)  \tag{lb}\\
& \frac{\rho_{2}}{\rho_{1}}=\frac{b}{b-1+a / D}, \theta_{2}=\varphi-\arccos \sqrt{ }\left(\frac{1}{1+\frac{\tan ^{2} \varphi}{b^{2}}\left(b-1+\frac{a}{D}\right)^{2}}\right)
\end{align*}
$$

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Fig. 1. Break decay in the interaction of a skew shock wave with the surface of contact of two media (a regular regime).

Since, for small angles of incidence $\varphi$, the flow behind the SSW remains subsonic, break decay (BD) is accompanied by formation of a reflected shock wave (RISW, Fig. 1). The parameters of medium 1 behind the RISW are

$$
\begin{gather*}
p_{3}=\frac{2 \rho_{2} q_{2}^{2}}{k+1} \sin ^{2}\left(\beta+\theta_{2}\right)-\frac{k-1}{k+1} p_{2}, \\
q_{3}=q_{2} \cos \left(\beta+\theta_{2}\right) \sqrt{1+\tan ^{2}\left(\beta+\theta_{2}\right) K^{2}\left(p_{3} / p_{2}\right)}, \frac{\rho_{2}}{\rho_{3}}=K\left(p_{\dot{3}} / p_{2}\right), \\
\theta_{3}=\arccos \sqrt{\left(\frac{1}{1+\tan ^{2}\left(\beta+\theta_{2}\right) K^{2}\left(p_{3} / p_{2}\right)}\right)-\beta,} \text {, } \tag{2a}
\end{gather*}
$$

for a PM and

$$
\begin{gather*}
p_{3}=p_{2}+\frac{\rho_{2} q_{2}^{2}}{b} \sin ^{2}\left(\beta+\theta_{2}\right)\left(1-\frac{a}{q_{2} \sin \left(\beta+\theta_{2}\right)}\right), \frac{\rho_{2}}{\rho_{1}}=\frac{b}{b-1+a /\left[q_{2} \sin \left(\beta+\theta_{2}\right)\right]} \\
q_{3}=q_{2} \cos \left(\beta+\theta_{2}\right) \sqrt{\left(1+\frac{\tan ^{2}\left(\beta+\theta_{2}\right)}{b^{2}}\left(b-1+\frac{a}{q_{2} \sin \left(\beta+\theta_{2}\right)}\right)^{2}\right)}  \tag{2b}\\
\theta_{3}=\arccos \sqrt{\left(\frac{1}{1+\frac{\tan ^{2}\left(\beta+\theta_{2}\right)}{b^{2}}\left(b-1+\frac{a}{q_{2} \sin \left(\beta+\theta_{2}\right)}\right)^{2}}\right)-\beta} .
\end{gather*}
$$

for an AM. The characteristics of medium 2 immediately behind the refracted shock wave (RrSW) are

$$
\begin{gather*}
p_{4}=\frac{2 \rho_{5} q_{1}^{2}}{m+1} \sin ^{2} \alpha-\frac{m-1}{m+1} p_{0}, \frac{\rho_{5}}{\rho_{4}}=M\left(p_{4} / p_{0}\right)=\frac{m+1+(m-1) p_{4} / p_{0}}{m-1+(m+1) p_{4} / p_{0}}, \\
q_{4}=q_{1} \cos \alpha \sqrt{1+\tan ^{2} \alpha M^{2}\left(p_{4} / p_{0}\right)}, \quad \theta_{4}=\alpha-\arccos \sqrt{ }\left(\frac{1}{1+\tan ^{2} \alpha M^{2}\left(p_{4} / p_{0}\right)}\right), \tag{3a}
\end{gather*}
$$

for a PM and

$$
\begin{gather*}
p_{4}=p_{0}+\frac{\rho_{5} q_{1}^{2}}{d} \sin ^{2} \alpha\left(1-\frac{c}{q_{1} \sin \alpha}\right) \\
q_{4}=q_{1} \cos \alpha \sqrt{\left(1+\frac{\tan ^{2} \alpha}{d^{2}}\left(d-1+\frac{c}{q_{1} \sin \alpha}\right)^{2}\right)} \\
\left.\frac{\rho_{4}}{\rho_{5}}=\frac{d}{d-1+c /\left[q_{1} \sin \alpha\right]}, \quad \theta_{4}=\alpha-\arccos \sqrt{\left(1+\frac{\tan ^{2} \alpha}{d^{2}}\left(d-1+\frac{c}{q_{1} \sin \alpha}\right)^{2}\right.}\right) \tag{3b}
\end{gather*}
$$

for an AM.
For simultaneous solution that would make it possible to determine the parameters of the flow resulting from the BD , system (1)-(3) yields 12 equations for calculating 14 unknowns: $p_{2}, q_{2}, \rho_{2}, \theta_{2}, p_{3}, q_{3}, \rho_{3}, \theta_{3}$, $p_{4}, q_{4}, \rho_{4}, \theta_{4}, \alpha$, and $\beta$. Here, $p_{2}, q_{2}, \rho_{2}$, and $\theta_{2}$ are expressed in terms of the initial values $p_{0}, q_{1}, \rho_{1}, D, \varphi$, and $k$ (or $a$ and $b$ ). The two equations that are missing to close the system can be obtained from the conditions of equality of the sures and the normal velocity component of the two media immediately behind the PC:

$$
\begin{equation*}
p_{3}=p_{4}, \quad q_{3} \sin \theta_{3}=q_{4} \sin \theta_{4} \tag{4}
\end{equation*}
$$

The first equation of (4) leads to an equality that relates $\alpha$ and $\beta$ :

$$
\begin{align*}
& \sin \alpha=\frac{c}{2 D} \sin \varphi\left[1+\sqrt{ }\left(1+4 d \frac{p_{3}-p_{0}}{\rho_{5} c^{2}}\right)\right]  \tag{5}\\
& \sin \alpha=\sin \varphi \sqrt{P M} \\
& \sin
\end{align*}
$$

The second equation of (4) yields a transcendental relation for determining $\beta$, and in accordance with the number of possible combinations of the surface it has three variants (for $\mathrm{PM}-\mathrm{PM}, \mathrm{AM}-\mathrm{AM}$, and $\mathrm{PM}-\mathrm{AM}$ ); however, there is little purpose in writing them explicitly here. Having found the numerical value of $\beta$ from the second equation of (4), we can calculate the remaining quantities of the BD after substitution of $\beta$ into the equations of the combined system (1)-(3) and conditions (5).

In $[1,2]$, it was established that two characteristic angles and their relation to $\varphi-$ the angle $\varphi_{\mathrm{t}}$ that corresponds to the situation where the BD is the simplest combination of flat SSW and RrSW and is called the angle of direct refraction in [2] and $\varphi_{c}$ determined as the angle at which supersonic flow immediately behind the SSW gives way to subsonic flow - have a determining effect on the process of interaction between the SSW and a flat SC. We find $\varphi_{\mathrm{c}}$ from the condition $q_{2} / c_{2}=1$. For $\varphi \geq \varphi_{c}$, the flow behind the SSW will be subsonic. For a PM

$$
\begin{equation*}
\varphi_{\mathrm{c}}=\operatorname{arcctan} \sqrt{ }\left(\left[\frac{2 k}{k+1}-\frac{k-1}{k+1} \frac{k p_{0}}{\rho_{1} D^{2}}-K\left(p_{2} / p_{0}\right)\right] K\left(p_{2} / p_{0}\right)\right) \tag{6a}
\end{equation*}
$$

and for an $\mathrm{AM}\left(x=\rho_{2} / \rho_{1}\right)$

$$
\begin{equation*}
\varphi_{c}=\operatorname{arcctan} \sqrt{ }\left(\left(\frac{a}{D}\right)^{2} \frac{(2 x-1)[b-(b-1) x]+2(b-1)(x-1) x}{[b-(b-1) x]^{3}}-\frac{1}{x^{2}}\right) \tag{6b}
\end{equation*}
$$



Fig. 2. Angles $\varphi_{1}$ and $\varphi_{c}(\mathrm{deg})$ vs. $D(\mathrm{~m} / \mathrm{sec}):$ a) $\varphi_{t}(D)$ for the SC: 1) air-krypton, 2) air-propane, 3) propane-krypton and $\varphi_{c}(D)$ for: 4) air, 5) propane; b) $\varphi_{c}(D)$ for: 1) aluminum, 2) iron, 3) lead, 4) copper, 5) titanium, 8) sand, 7) water, 8) ethyl alcohol, 9) beryllium, 10) paraffin; c) $\varphi_{1}(D)$ for the SC: 1) air-iron, 2) air-aluminum, 3) air-sand, 4) air-water; d) $\varphi_{\mathrm{t}}(D)$ for the SC: 1) titanium-cobalt, 2) titanium-iron, 3) titanium-lead; $\varphi_{\mathrm{c}}(D)$ for titanium (4); e) $\varphi_{1}(D)$ for the SC: 1) water-iron, 2) water-sand; $\varphi_{c}(D)$ for water (3); f) $\varphi_{1}(D)$ for the $S C:$ 1) aluminum-iron, 2) aluminum-titanium; $\varphi_{c}(D)$ for aluminum (3).

The dependence $\varphi_{c}(D)$ for certain gases is given in Fig. 2a; in Fig. 2b, it is plotted for certain materials that differ in quality. The angle of direct refraction can be calculated in solving systems (1) and (3) simultaneously when $\varphi$ $=\varphi_{\mathrm{t}}$ and

$$
\begin{equation*}
p_{2}=p_{4}, \quad q_{2} \sin \theta_{2}=q_{4} \sin \theta_{4} . \tag{7}
\end{equation*}
$$

The first equation of (7) determines the relationship between $\varphi_{\mathrm{t}}$ and $\alpha$ in the case where medium 2 is a PM and an AM, respectively:

$$
\begin{align*}
& \sin \alpha=\sin \varphi_{\mathrm{t}} \sqrt{ }\left(\frac{p_{2}(m+1)+p_{1}(m-1)}{2 \rho_{5} D^{2}}\right),  \tag{8}\\
& \sin \alpha=\sin \varphi_{1}\left(1+\sqrt{ }\left(1+4 d \frac{p_{2}-p_{0}}{\rho_{5} c^{2}}\right)\right) .
\end{align*}
$$



Fig. 3. Scheme of the break decay in the interaction of an SSW with the SC for: a) RDR; 6) SIR; c) SSIR; d) WIR.

The second equality of (7), in view of (8), yields a transcendental equation in the single parameter $\varphi_{t}$. Data for a number of gas-gas pairs of substances are given in Fig. 2a, for a surface of contact of gas-solid type in Fig. 2c, and for a surface of contact of AM-AM type in Fig. 2d, e, and f. It can easily be noted that for a boundary of contact of gas-dense medium type $\varphi_{t}$ is close $\pi / 2$, and $\varphi_{t}$ certainly always exceeds $\varphi_{c}$.

Calculations of [1, 2] establish a clear sequence of change in interaction regime as a function of the two characteristic angles. First of all, for all $\varphi<\varphi_{t}$ and $\varphi<\varphi_{c}$, there exists a regime of regular reflection, whatever the relationship between $\varphi_{\mathrm{t}}$ and $\varphi_{\mathrm{c}}$. For all remaining $\varphi$, several regimes of interaction [2] classified as irregular regimes are known:

1) when $\varphi_{c}>\varphi_{1}$ (Fig. 2 shows that this regime is realizable for SCs of relatively homogeneous media whose densities and acoustic impedances are similar) there are three kinds of irregular interaction: a) $\varphi=\varphi_{t}$ is a regime of direct refraction (RDR) (Fig. 3a); it is possible for any type of SC; b) $\varphi_{t}<\varphi<\varphi_{c}$ is a strong irregular regime (SIR) [1] with a three-wave configuration that is elevated above the SC and a Mach wave (MW) entering it whose convexity faces the incoming flow while the center of curvature is to the left of the PC and above the tangential break (TB) (Fig. 3b); judging from Fig. 2, the SIR is characteristic of gases in the region of high values of $D$ and of surfaces of contact of AM-AM type in the region of low values of $D$; c) $\varphi \geq \varphi_{c}$ is strong shockless irregular reflection (SSIR) [1]; the MW orientation is the same as for point 1 b ; however the three-wave configuration disappears since a shock regime behind an SSW is impossible (Fig. 3c); this regime is inherent to gases in the region of high velocities and to all SCs at large angles of incidence that are close to a right angle;
2) for $\varphi_{c} \leq \varphi_{t}$ (the regime characteristic of gases in the region of small $D$ and of dense substances in the region of large $D$ ) (Fig. 2) there also are three kinds of irregular reflection: a) $\varphi_{c}<\varphi<\varphi_{t}$ is a regime of weak irregular reflection (WIR) with an MW whose convexity points away from the incoming flow; the center of MW curvature is to the right of the PC under the SC (Fig. 3d); it is typical of the majority of the angles of irregular reflection for surfaces of gas-dense substance type (Fig. 2) and gas-gas type in the region of low $D$; b) $\varphi=\varphi_{1}$ is an RDR; it is analogous to point 1a; c) $\varphi>\varphi_{1}$ is SSIR; it coincides with point 1c and is characteristic of all the contact surfaces at angles $\varphi \rightarrow \pi / 2$.

An MW formed in irregular reflection is described by a curve with two characteristic angles [1, 2 ]: the upper angle $v$ is the angle of entry of the MW into the triple point if one exists (if there is no triple point, $v \equiv \varphi$ ),
and the lower angle $\mu$ is the angle of entry of the MW into the PC. A procedure for determining these angles was described in [1, 2]. To calculate $\mu$, we need to obtain the simultaneous solution of (4) and

$$
\begin{gather*}
p_{\mathrm{m}}=\frac{2 \rho_{1} q_{1}^{2}}{k+1} \sin ^{2} \mu-\frac{k-1}{k+1} p_{0}, \frac{\rho_{1}}{\rho_{\mathrm{m}}}=K\left(p_{\mathrm{m}} / p_{0}\right) \\
q_{\mathrm{m}}=q_{1} \cos \mu \sqrt{1+\tan ^{2} \mu K^{2}\left(p_{\mathrm{m}} / p_{0}\right)}, \theta_{\mathrm{m}}=\mu-\arccos \sqrt{\left(\frac{1}{1+\tan ^{2} \mu K^{2}\left(p_{\mathrm{m}} / p_{0}\right)}\right)} \tag{9a}
\end{gather*}
$$

if the type of medium 1 is a PM [1], or

$$
\begin{gather*}
p_{\mathrm{m}}=p_{0}+\frac{\rho_{1} q_{1}^{2}}{b} \sin ^{2} \mu\left(1-\frac{a}{q_{1} \sin \mu}\right) \\
q_{\mathrm{m}}=q_{1} \cos \mu \sqrt{\left(1+\frac{\tan ^{2} \mu}{b^{2}}\left(b-1+\frac{a}{q_{1} \sin \mu}\right)^{2}\right)} \\
\left.\frac{\rho_{\mathrm{m}}}{\rho_{1}}=\frac{b}{b-1+a /\left[q_{1} \sin \mu\right]}, \quad \theta_{\mathrm{m}}=\mu-\arccos \sqrt{\left(\frac{\tan ^{2} \mu}{b^{2}}\left(b-1+\frac{a}{q_{1} \sin \mu}\right)^{2}\right.}\right) \tag{9b}
\end{gather*}
$$

for an AM. The conditions on the SC

$$
\begin{equation*}
p_{\mathrm{m}}=p_{4}, \quad q_{\mathrm{m}} \sin \theta_{\mathrm{m}}=q_{4} \sin \theta_{4} \tag{10}
\end{equation*}
$$

close the system obtained. As before, the first equation of (10) establishes a relationship between $\alpha$ and $\mu$ that is analogous to (5) with substitution of $\rho_{\mathrm{m}}$ for $\rho_{3}$. The second equation is transformed to a transcendental relation for calculating the angle $\mu$. Its numerical solution for $\mu$ enables us to calculate the parameters of the flow at the very base of the "Mach foot" in medium 1 and behind the $\operatorname{RrSW}$ in medium 2. It is necessary to determine the parameters of the upper angle $v$ only in cases of irregular reflection where $\varphi<\varphi_{c}$ and a three-wave configuration exists. In all remaining cases, $\nu \equiv \varphi$ and the MW changes smoothly to an SSW. To find $\nu$, we need to obtain the simultaneous solution of (1), (2), and

$$
\begin{gather*}
p_{v}=\frac{2 \rho_{1} q_{1}^{2}}{k+1} \sin ^{2} v-\frac{k-1}{k+1} p_{0}, \frac{\rho_{1}}{\rho_{\nu}}=K\left(p_{v} / p_{0}\right) \\
q_{\nu}=q_{1} \cos v \sqrt{1+\tan ^{2} v K^{2}\left(p_{v} / p_{0}\right)}, \theta_{v}=v-\arccos \sqrt{\left(\frac{1}{1+\tan ^{2} v K^{2}\left(p_{v} / p_{0}\right)}\right)} \tag{11a}
\end{gather*}
$$

for a PM [1, 2] or

$$
\begin{gathered}
p_{v}=p_{0}+\frac{\rho_{1} q_{1}^{2}}{b} \sin ^{2} v\left(1-\frac{a}{q_{1} \sin v}\right) \\
q_{v}=q_{1} \cos v \sqrt{\left(1+\frac{\tan ^{2} v}{b^{2}}\left(b-1+\frac{a}{q_{1} \sin v}\right)^{2}\right)},
\end{gathered}
$$



Fig. 4. Angles $\varphi_{\mathrm{c}}$ and $\theta_{\mathrm{c}}$ (deg) vs. $D(\mathrm{~m} / \mathrm{sec})$ : a) $\varphi_{\mathrm{c}}(D)$ for: 1) iron, 2) lead; $\theta_{c}(D)$ for: 3) iron, 4) lead; b) $\varphi_{c}(D)$ for: 1) water, 2) sand; $\theta_{c}(D)$ for: 3) water, 4) sand.

$$
\begin{equation*}
\theta_{\nu}=v-\arccos \sqrt{\left(\frac{1}{1+\frac{\tan ^{2} v}{b^{2}}\left(b-1+\frac{a}{q_{1} \sin v}\right)^{2}}\right), \frac{\rho_{\nu}}{\rho_{1}}=\frac{b}{b-1+a^{\prime}\left[q_{1} \sin v\right]}} \tag{11b}
\end{equation*}
$$

for an AM. These three systems offer 12 equations for finding 14 unknowns: $p_{2}, q_{2}, \rho_{2}, \theta_{2}, p_{3}, q_{3}, \rho_{3}, \theta_{3}$, $p_{\nu}, q_{\nu}, \rho_{\nu}, \theta_{\nu}, \nu$, and $\beta$. The two missing relations follow from the conditions of equality of the pressure and the normal component of the flow velocity on the two sides of the TB:

$$
\begin{equation*}
p_{2}=p_{\nu}, \quad q_{2} \sin \theta_{2}=q_{\nu} \sin \theta_{\nu} . \tag{12}
\end{equation*}
$$

The first equality of (12) determines the relationship between $\nu$ and $\beta$, while the second equation, upon the corresponding substitutions, permits numerical determination of $\beta$ and subsequently of all parameters of the flow near the triple point. The rotation of the SC behind the PC is prescribed by the angle

$$
\begin{equation*}
\lambda=\arctan \frac{q_{4} \sin \theta_{4}}{q_{1}} \tag{13}
\end{equation*}
$$

the rotation of the $T B$ is prescribed by

$$
\begin{equation*}
\tau=\arctan \frac{q_{\nu} \sin \theta_{\nu}}{q_{1}} \tag{14}
\end{equation*}
$$

for an SIR (Fig. 3b) and by

$$
\begin{equation*}
\tau=\arctan \frac{q_{2} \sin \theta_{2}}{q_{1}} . \tag{15}
\end{equation*}
$$

for all remaining cases.
Of special interest is the BD in an arbitrary substance on a rigid surface (RS). For gases, this problem was considered in [2] in sufficient detail, and therefore we investigate the interactions of an SSW that propagates in an AM in contact with an RS. The flow behind the SSW is described by system (1b) while the flow behind the RISW is described by ( 2 b ). To determine nine unknowns ( $p_{2}, q_{2}, \rho_{2}, \theta_{2}, p_{3}, q_{3}, \theta_{3}$, and $\beta$ ), the combined system (1b) and (2b) offers eight equations. The condition of rigidity of the wall that requires that the flow near the RS be parallel to the surface, i.e., $\theta_{3}=0$, closes the system, which leads to a relation for determining $\beta$ :

$$
\begin{equation*}
\cos ^{2} \beta\left[1+\frac{\tan ^{2}\left(\beta+\theta_{2}\right)}{b^{2}}\left\{b-1+\frac{a}{q_{2} \sin \left(\beta+\theta_{2}\right)}\right\}\right]^{2}=1 . \tag{16}
\end{equation*}
$$

Having solved transcendental equality (16), we can determine numerically all the parameters of the developed flow. Clearly, as soon as $\varphi \geq \varphi_{c}$, the regime of regular reflection gives way to the irregular regime since the RISW cannot exist. However, calculations show that the regular regime can give way to the irregular regime somewhat earlier. The matter is that a solution of (16) in the region of real $\beta$ is possible only for angles $\varphi \leq \theta_{c}$, where $\theta_{c}$ is defined [3] as the angle of change in the reflection regime from regular to irregular. In the case where $\theta_{c}<\varphi_{c}$, for $\varphi=$ $\theta_{c}$ the regular regime of reflection is replaced by an irregular regime with a RISW and the triple point elevated above the SC. The dependences $\theta_{c}(D)$ and $\varphi_{\mathrm{c}}(D)$ for various substances are given in Fig. 4a and b. It should be borne in mind that these dependences are not always suitable for weak SWs, since in these cases the hydrodynamic theory of shock waves is inapplicable for solids. In cases where $\theta_{c}>\varphi_{c}$, the angle $\theta_{c}$ has no effect on the BD because it is determined by the presence of a rigid wall but fails to reflect the inherent regularities of the construction of the shock-wave configuration. By the time $\varphi$ attains the value of $\theta_{c}$ (for the case of $\varphi_{c}<\theta_{c}$ ) regular reflection, whose disappearance is announced by $\theta_{\mathrm{c}}$, is already absent (since the RISW cannot develop). The second angle $\varphi_{1}$ that is characteristic of reflection processes can be calculated from the condition of construction of a straight one-wave (the lower medium is rigid) configuration. Indeed, the conditions behind the SSW front are prescribed by the solution of system ( 1 b ), which contains four equations for finding five unknowns: $p_{2}, q_{2}, \rho_{2}, \theta_{2}$, and $\varphi_{1}$. The condition on the RS that requires parallelism of the flow and the surface, $\theta_{2}=0$, closes the system. This is possible for two values of $\varphi_{\mathrm{t}}: 0$ and $\pi / 2$. However, for $\varphi_{\mathrm{t}}=0$, the parameters behind the SSW lose physical meaning, and hence $\varphi_{\mathrm{t}}=\pi / 2$, i.e., in interaction between the SSW and the RS the convexity of the MW is always pointed away from the incoming flow, the center of curvature of the MW is to the right of the PC, and so-called weak regimes of reflection will be realized. If $\theta_{c}<\varphi_{c}$, there is weak shock irregular reflection (WSIR) for all $\theta_{c}<\varphi<\varphi_{c}$ [2]; as soon as $\varphi \geq \varphi_{c}$, it is replaced by a regime of weak irregular reflection (WIR) [2]. To solve the problem, it remains only to establish the angles that bound the curve of the MW. The angle $\mu$ for both WSIR and WIR is found in solving system ( 9 b ); the four equations of the system for finding five unknowns ( $p_{\mathrm{m}}, q_{\mathrm{m}}, \rho_{\mathrm{m}}, \theta_{\mathrm{m}}$, and $\mu$ ) are completed by the condition of wall rigidity, i.e., $\theta_{\mathrm{m}}=0$, which yields that $\mu=0$ or $\pi / 2$. The parameters of the medium behind the MW lose physical meaning when $\mu=0$, and consequently, $\mu=\pi / 2$ (the center of curvature of the MW always lies on the RS). For WIR, this solves the problem, since $\nu \equiv \varphi$, and the MW changes to an SSW smoothly. For WSIR, we need, using the simultaneous solution of systems (1), (2), and (11b) with condition (12), to determine the angles $\nu$ and $\beta$. For example, for water, when $\varphi=48^{\circ}$ and $D=2500 \mathrm{~m} / \mathrm{sec}$ the angle $\nu$ is approximately $83.9^{\circ}$ and the angle $\beta$ is approximately $27.7^{\circ}\left(\varphi_{\mathrm{t}} \approx 47^{\circ}\right.$ and $\varphi_{\mathrm{c}} \approx 49.5^{\circ}$ ).

Thus, the general picture of irregular reflection is fully specified by the relation of the angle of incidence to the angle of total refraction and the angle of change in the regime of the flow from supersonic to subsonic, which are determined for a given pair of interacting materials in a wide range of contacting media that belong to various classes of substances.

## NOTATION

$D$, shock-wave velocity; $c$, velocity of sound in the medium (always with a subscript); $p$, pressure; $q$, total velocity of the flow in a coordinate system connected with the point of intersection of the skew shock wave and the surface; $k$ and $m$, polytrope indices of the gas for medium 1 and medium 2 , respectively; $a, b$ and $c, d$, coefficients of the $D-U$ adiabats for medium 1 and medium 2 , respectively ( $c$ here is always without a subscript); $K$ and $M$, functions; $\rho$, density; $\alpha$, slope of the refracted shock wave to the SC; $\varphi$, slope of the skew wave to the SC; $\lambda$, angle of rotation of the SC behind the PC; $\tau$, angle of rotation of the tangential break; $\mu$, angle of entry of the Mach wave into the point of contact of the skew shock wave with the SC; $\nu$, angle of the Mach wave at the triple point; $\theta$, angle of rotation of the vector of the total velocity of the flow behind the shock-wave front; $\vartheta$, angle of opening of the sector of a circumference that approximates the Mach-wave surface; $\theta_{c}$, critical angle at which supersonic flow behind the skew shock wave is replaced by subsonic flow. Subscripts and superscripts: 1, initial parameters of medium $1 ; 2$, parameters of medium 2 behind the skew shock wave; 3 , parameters of medium 1 behind the reflected shock wave; 4 , parameters of medium 2 behind the refracted shock wave; 5 , initial parameters of medium $2 ; \mathfrak{t}$,
transition angle between regimes of irregular and regular interaction; $m$, parameters of the gas in the lower part of the Mach wave at the angle $\mu ; \nu$, parameters of the gas in the upper part of the Mach wave at the angle $\nu$.

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